Laplace's method

Suppose f(x) is a twice continuously differentiable function on [a, b], and there exists a unique point $x_0 \in (a, b)$ such that:

$$f(x_0) = \max_{x \in [a,b]} f(x)$$
 and $f''(x_0) < 0$.

Then:

$$\lim_{n \to \infty} \frac{\int_{a}^{b} e^{nf(x)} \, dx}{e^{nf(x_0)} \sqrt{\frac{2\pi}{n(-f''(x_0))}}} = 1.$$
(1)

Euler Product Formula

Let's take $s \in \mathbb{C}$. The Euler Product Formula, when $\Re(s) > 1$, is given by:

$$\prod_{p \in \mathbb{P}} \left(\frac{1}{1 - p^{-s}}\right) = \prod_{p \in \mathbb{P}} \left(\sum_{k=0}^{\infty} \frac{1}{p^{ks}}\right) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} \, \mathrm{d}x \tag{2}$$

Where:

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} \,\mathrm{d}x$$

Stirling's formula

It is also called Stirling's approximation for factorials:

$$\lim_{n \to +\infty} \frac{n!}{\sqrt{2\pi n} \left(n / e\right)^n} = 1$$
(3)

Also frequently written as:

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

One can easily derive the following limit from Stirling's formula:

$$\lim_{n \to \infty} \frac{\left(n!\right)^{1/n}}{n} = \frac{1}{e}$$